

HW 3

Assigned 02/07, Due 02/19

Q1. Show that the following properties of distributed knowledge are valid:

- (a) $(D_G\phi \wedge D_G(\phi \Rightarrow \psi)) \Rightarrow D_G\psi$
- (b) $D_{\{i\}}\phi \Leftrightarrow K_i\phi$
- (c) $D_G\phi \Rightarrow D_{G'}\phi$ if $G \subseteq G'$
- (d) $D_G\phi \Rightarrow D_G D_G\phi$

Q2.

- Prove that $(\mathcal{I}, r, m) \models \Diamond\phi$ iff $(\mathcal{I}, r, m) \models \text{true}U\phi$, and $(\mathcal{I}, r, m) \models \Box\phi$ iff $(\mathcal{I}, r, m) \models \neg\Diamond\neg\phi$.
- Construct an interpreted system \mathcal{I} and two points (r, m) and (r', m') in \mathcal{I} such that $r(m) = r'(m')$, but $(\mathcal{I}, r, m) \models \Diamond p$ and $(\mathcal{I}, r', m') \models \neg\Diamond p$

Q3. Show that knowledge of ignorance does not necessarily persist in systems where agents have perfect recall. In particular, construct an interpreted system \mathcal{I} where agents have perfect recall and a run r in \mathcal{I} such that $(\mathcal{I}, r, o) \models \neg K_1 p$ and $(\mathcal{I}, r, m) \models K_1 p$ for $m > 0$.

Q4. A formula ϕ is said to be *stable* (with respect to the interpreted system \mathcal{I}) if once ϕ is true it remains true; i.e., if we have $\mathcal{I} \models \phi \Rightarrow \Box\phi$. Assume that ϕ_1 and ϕ_2 are stable.

- (a) Show that $\phi_1 \wedge \phi_2$ and $\phi_1 \vee \phi_2$ are stable.
- (b) If, in addition, \mathcal{I} is a system where agents have perfect recall, show that $K_i\phi_1$ and $C_G\phi_1$ are stable. Thus, in a system where agents have perfect recall, if an agent knows a stable formula at some point, then he knows it from then on.
- (c) Show that if we assume in addition that the system is synchronous, then $D_G\phi_1$ is stable.