HW 2
Assigned 01/29, Due 02/07

Q1. Describe how three agents might negotiate to find a common telephone line for a conference call. Assume that Agent A has telephone lines 1,2,3; Agent B, 1,3; and Agent C, 2,3.

The negotiation proceeds pairwise: two agents at a time. The agents negotiate in order: $A, B, C, A, B, C, A, \cdots$. Also, alternate lines are chosen in the order specified above for each agent.

Initially,
Agent A proposes line 1 to Agent B, and agent B accepts it.
Agent B proposes line 1 to Agent C, but agent C rejects it.
Complete the process until all agents have picked a common line.

Q2. The *wise men puzzle* is a well-known variant of the muddy children puzzle. The standard version of the story goes as follows: There are three wise men. It is common knwoledge that there are three red hats and two white hats. The king puts a hat on the head of each of the three wise men, and asks them (sequentially) if they know the color of the hat on their head. The first wise man says that he does not know; the second wise man says that he does not know; then the third wise man says that he knows.

   (a) What color is the third wise man's hat?

   (b) We have implicitly assumed in the story that the wise men can all see. Suppose we assume instead that the third wise man is blind and that it is common knowledge that the first two wise men can see. Can the third wise man still figure out the color of his hat?

Q3. Show that if $M \vDash \phi \Rightarrow \psi$, then

   (a) $M \vDash K_i \phi \Rightarrow K_i \psi$,

   (b) $M \vDash C_G \phi \Rightarrow C_G \psi$.

Q4. Prove the following assuming that the $\mathcal{K}_i$ relations are equivalence relations:

   (a) $\vDash \neg \phi \Rightarrow K_i \neg K_i \phi$,

(b) $\vDash \neg\phi \Rightarrow K_{i_1} \cdots K_{i_k} \neg K_{i_k} \cdots K_{i_1} \phi$ for any sequence $i_1, \cdots, i_k$ of agents.

Q5. Read the paper by Mike Wellman on computational market models (you can find it on the reading list on the webpage) and write a 2-page long article explaining the mathematical model used and the main results of the paper.