Lecture 5
Formal Methods in DAI

E. Haghverdi

January 22, 2002

We continue our discussion of the macro level. As was mentioned before there is no central control in a multiagent system and the fact that information and control is distributed makes the maintenance of global coherence a very difficult challenge. This problem is addressed by coordination protocols that we will not discuss. However, we will look at two manifestations of coordination, namely *cooperation* and *competition.*

Cooperation protocols use the basic strategy of *divide and conquer*. This involves *task decomposition* and *task distribution.* Task decomposition can be *spatial*, that is based on information sources and/or decision points or *functional* that is , based on expertise. The following task distribution mechanisms have been in use;

- **Contract Nets**: A contract net solves the so called *connection problem* when an appropriate agent is to be found for a task to be completed. The manager is the agent with some task he wants to be completed and agents that might have the capabilities to perform this task are called *potential contractors.* Manager announces the task and agents bid for it based on several criteria, e.g. their time, resource, activity level at that time, etc. Manager then evaluates the bids and offers the task to one of the agents. This agent in turn may need to become manager and try to hire other agents to complete parts of the task he has to complete.
- **Market Mechanisms**: This will be in HW2 based on the paper by Mike Wellman. The idea though is that market equilibrium corresponds to an optimal resource allocation.
- **Multiagent Planning**: planning agents have the responsibility for task assignment.
- **Organizational Structure**: agents have fixed responsibilities for particular tasks.

Next we look at an example in the case of competitive situation, where we have agents with conflicting goals or self-interested agents only.

Negotiation

A negotiation protocol is a process by which agents come to an agreement while pursuing their individual goals. One of the most successful treatments of such protocols has been the work by Rosenschein and Zlotkin from Hebrew University, written up in the book called Rules of Encounter. We will have parts of the book summarized over the homework assignments together with related problems. This will prepare us for the discussion on mechanism design and auction theory during the last 3-4 weeks of this class.

1 Formal Methods in DAI

There are two different application areas for formal methods in multiagent systems, namely:

- internal specification language to be used by the agents in their reasoning and decision making, like the vacuum cleaning example, the main technique used here is theorem proving,
- external metalanguage to be used by the designer to specify and verify agent behavior. The main techniques here are theorem proving and model checking.

We will, for the most part of the course be concerned with logical techniques. Clearly formal techniques are not limited to logical ones only, as we will see towards the end of the course. Several logical formalisms, all based on propositional and predicate logic have been studied and applied in the context of MAS:

- Modal logic, where different modes of truth like necessarily true and possibly true are studied
- Dynamic logic, modal logic of action: necessity and possibility operators are based on the actions available.
- Temporal logic, logic of time: one might assume that time is linear vs branching, discrete vs dense, etc.
- Modal logics of knowledge, belief, desire and other cognitive primitives.

We will study logic of knowledge in more detail.

2 Logic of Knowledge

The study of knowledge and its properties goes back to Aristotle and early Greek philosophers, however the logical analysis of knowledge can be traced back to von Wright's work in 1951 and the first book-length treatment of Hintikka in 1962, called Knowledge and Belief. Also *common knowledge* was defined by the philosopher D. Lewis in 1969 and introduced to economics by R. Aumann in 1976. For more details about history see [K].

2.1 The Muddy Children Puzzle

Suppose there are n children playing and are told by their mother that they should not get dirty lest there will be severe consequences. k of them get mud on their foreheads. Along comes the father announcing: "At least one of you has mud on your forehead." He then repeatedly asks the question "Does any of you know whether you have mud on your forehead?" The puzzle is: will it be possible for the muddy kids to so conclude after finitely many questions by their father?

Observations: We discussed the proof of the following propositions in class.

Proposition 2.1 *All children will answer "No" to father's first $k-1$ questions.*

Proposition 2.2 *The muddy children will answer "Yes" to father's kth question.*

Let p stand for the statement "At least one child has mud on his forehead" and Ep denote that everybody knows p. Then

Proposition 2.3 *If father does not announce p. No matter what k is and what the number of questions asked by father is, the muddy children will not be able to conclude that they have mud on their foreheads.*

Proposition 2.4 *With k muddy children $E^{k-1}p$ is true and $E^k p$ is not. Also $E^k p$ suffices for muddy children to conclude that they have mud on their foreheads.*

Father's announcement thus makes $E^{k-1}p$ into common knowledge of p. This example shows the subtleties of reasoning about others' knowledge and a clear need for a formal approach to this type of reasoning. We introduce modal knowledge operators on top of propositional logic to deal with this.

2.2 The Logic

We define the syntax first. Let Φ denote the set of atomic propositions and suppose there are n agents. The well-formed formulas are defined in BNF:

$$\phi ::= p|\phi \wedge \phi|\neg\phi|K_i\phi$$

Here $p \in \Phi$ and $i \in \{1, \cdots, n\}$. $K_i\phi$ means that agent i knows ϕ. We can express possibility (dual of knowledge) as follows: agent i considers ϕ possible if it does not know not ϕ, that is if $\neg K_i \neg \phi$. Thus, the statement "Agent i does not know whether ϕ" gets translated as

$$\neg K_i \neg \phi \wedge \neg K_i \phi.$$

Also note that other connectives (disjunction, implication) and logical constants (true and false) can be defined using conjunction and negation.