Math 670
Homework 3
Assigned 10/12, Due 10/23

1. Write a sequent calculus proof in CL for $(A \longrightarrow B) \longrightarrow (A \longrightarrow (A \longrightarrow B))$
2. Write a sequent calculus proof in CL for the Pearce's Law: $((A \longrightarrow B) \longrightarrow A) \longrightarrow A$.
3. Show that Pearce's Law is not provable in IL.
4. Prove the formulas $(A \otimes (A \multimap B)) \multimap B$ and $A^{\perp} \parr A$. Show that $(A \otimes (A \multimap B) \otimes (A \multimap C)) \multimap B \otimes C$ and $(A \otimes A \otimes A \otimes (A \multimap B) \otimes (A \multimap C)) \multimap B \otimes C$ are not provable in CLL?
5. Prove the following linear equivalences

$$!(A\&B) \equiv (!A) \otimes (!B) \quad ?(A \oplus B) \equiv (?A) \parr (?B).$$

6. Prove that $!!A$ is linearly equivalent to $!A$.

Extra Credit Prove that there are only seven modalities (combinations of two modalities ! and ?) up to linear equivalence; draw the lattice of derivability.