Math 670
Homework 2

Assigned 09/21, Due 10/02

1. If G is a group considered as a category, prove that a natural tranformation on the identity functor of G is just an element of the center of G.

2. Prove that a contravariant representable functor maps an epimorphism to a monomorphism.

3. Consider a functor $F : \mathcal{D} \longrightarrow \mathcal{C}$ and an object C of $\mathcal{C}$ Let $\mathcal{K}_C : \mathcal{D} \longrightarrow \mathcal{C}$ be the constant functor on C. Prove that a cone on F is just a natural transformation $\mathcal{K}_C \Rightarrow F$.

4. Let **Rel** be the category of sets and (binary) relations. Show that this indeed is a category. Prove that all finite products and coproducts exist.

5. For functors $S, T : \mathcal{C} \longrightarrow \mathcal{P}$ where $\mathcal{C}$ is a category and $\mathcal{P}$ is a preorder considered as a category, show that there is a natural transformation $S \Rightarrow T$ (which is then unique) if and only if $SC \leq TC$ for every object C in $\mathcal{C}$.

6. Let $(G_i)_{i \in I}$ be a family of abelian groups. Prove that their coproduct is given by the definition given in class.

7. Consider the category with objects (X, e, t), where X is a set, $e \in X$ and $t : X \longrightarrow X$ and with arrows $f : (X, e, t) \longrightarrow (X', e', t')$ the functions $f : X \longrightarrow X'$ such that $f(e) = e'$ and $ft = t'f$. Prove that this category has an initial object in which X is the set of natural numbers including 0, $e = 0$ and t is the successor function ($t(n) = n+1$).