Math 670
Homework 1

Assigned 09/07, Due 09/20

1. Prove in detail that the collection of topological spaces and continuous maps constitutes a category, denoted, **Top**.

2. Show that any poset $(X, \leq)$ can be viewed as a category $\mathcal{X}$. Provide all the details.

3. Verify that the power set functor $\mathcal{P}$ and the covariant hom functor $\mathcal{C}(C, -)$ for a category $\mathcal{C}$ and an object C in $\mathcal{C}$ are functors.

4. Discuss full, faithful, embedding and isomorphism properties for the functors in the previous problem.

5. Complete the details for naturality of the transformations $\sigma : Id \Rightarrow \mathcal{P}$ and $\mathcal{A}(f, -) : \mathcal{A}(B, -) \Rightarrow \mathcal{A}(A, -)$ for a category $\mathcal{A}$, objects A, B in $\mathcal{A}$ and $f : A \longrightarrow B$.

6. Consider the statement of the Yoneda Lemma discussed in class. Prove the naturality of the bijections $\theta_{F,A}$ in A.

7. Show that epimorphisms in **Top** are surjective continuous maps.

8. Show that monomorphisms in **Gr** are injective group homomorphisms.